Neutrino mass matrices with one texture zero and a vanishing neutrino mass

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Assuming Majorana nature of neutrinos, we investigate the singular one texture zero neutrino mass matrices in the flavor basis. We find that for the normal mass ordering with $m_1 = 0$, all the six one texture zero classes are now ruled out at 3σ confidence level, whereas for inverted mass ordering with $m_3 = 0$ only four classes out of total six can accommodate the latest neutrino oscillation data at 3σ confidence level. Moreover, only two classes can accommodate the present data at 1σ confidence level. We examine the phenomenological implications of the allowed classes for the effective Majorana mass, Dirac and Majorana CP-violating phases. Working within the framework of type-I seesaw mechanism, we present simple discrete Abelian symmetry models leading to all the phenomenologically allowed classes.

I.   INTRODUCTION

In the last three years, T2K, MINOS, Double Chooz, Daya Bay and RENO experiments [1–5] have established a non-zero and relatively large value of the reactor mixing angle θ_{13}. The result has further motivated experimentalists to pin down the long-standing problem of CP violation and mass ordering in neutrino sector. Recent global fits of neutrino oscillations [6, 7], have reported best fit points and 1σ errors on the Dirac CP-violating phase δ. On the theoretical side, in order to explain the pattern of neutrino masses and mixing, several ideas have been proposed in the literature which reduce the number of free parameters in neutrino mass matrix, e.g. some elements of the neutrino mass matrix are considered to be zero [8–11] or equal [12] or both the possibilities are taken together [13]. Similarly, some co-factors of the neutrino mass matrix are considered to be zero [14] or equal [12] or both [15]. The analysis of two texture zero neutrino mass matrices in the flavor basis restricts the number of experimentally compatible classes to seven. The phenomenological implications of one texture zero neutrino mass matrices have also been investigated in the literature [16–18] and it has been found that all the six possible classes with one texture zero in the neutrino mass matrix are experimentally viable.

In the flavor basis where the charged lepton mass matrix is diagonal, the Majorana neutrino mass matrix, being complex symmetric, contains six independent entries. If one of the elements is assumed to be zero, then we have six possible one texture zero classes which are shown in Table I. The condition of a single texture zero in the neutrino

P_1	P_2	P_3
$\begin{pmatrix} 0 & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$
P_4	P_5	P_6
$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & \times & 0 \\ \times & 0 & \times \end{pmatrix}$

TABLE I. Possible structures of neutrino mass matrices having one texture zero. $\times$ denotes the non-zero elements.

mass matrix is less restrictive (and hence less predictive) than the condition of two texture zeros. However, one can further reduce the number of free parameters of one texture zero Majorana neutrino mass matrices by considering one of the neutrino masses to be zero, which is still an experimentally viable scenario. In Refs. [17, 19], particular attention has been paid to neutrino mass matrices with one texture zero and a vanishing neutrino mass, which is

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termed as singular one texture zero model. Recently, individual classes of one texture zero with a vanishing neutrino mass have been studied by some authors, e.g. in Ref. [20] the authors have considered classes P_4 and P_5 with the additional constraint of a new kind of constrained sequential dominance (CSD2) [20, 21]. In Ref. [22] classes P_1, P_4 and P_5 have been explored with the motivation of linking the ratio of mass-squared differences and the (1,3) element of the neutrino mixing matrix. Class P_4 has also been studied in Ref. [23] and class P_3 has been studied in Ref. [24]. In Ref. [25] all the singular one texture zero classes along with other new texture structures have been obtained by systematically scanning the zeros of the Dirac and the right-handed Majorana neutrino mass matrices within the context of type-I seesaw mechanism [26].

Classes P_4, P_5 and P_6 have also been derived in Ref. [27] in the minimal type-I seesaw model considering only two right-handed heavy Majorana neutrinos. Further, these minimal models have been reinvestigated in Ref. [28] by keeping minimum number of parameters which can lead to successful leptogenesis and considering the relatively large value of θ_{13}. Recently, the renormalization group effects on neutrino mixing parameters corresponding to classes P_4, P_5 and P_6 have been considered in Ref. [29].

In view of the refined measurement of reactor mixing angle θ_{13} and with the motive of carrying out a complete phenomenological analysis of all the classes of singular one texture zero, we investigate neutrino mass matrices with one texture zero and a vanishing neutrino mass. Working within the framework of type-I seesaw mechanism [26], we construct simple neutrino mass models based on Z_8 discrete symmetry which lead to the experimentally allowed classes studied in the present work.

The rest of the paper is structured as follows: In section 2, we discuss the methodology employed to obtain the constraint equations for one texture zero. Section 3 is devoted to numerical analysis. In section 4, we give the details of symmetry realization of all the allowed classes. In section 4, we summarize our work.

II. METHODOLOGY

The effective Majorana neutrino mass matrix (M_ν) contains nine parameters which include three neutrino masses (m_1, m_2, m_3), three mixing angles (θ_{12}, θ_{23}, θ_{13}) and three CP-violating phases (δ, ρ, σ). In the flavor basis, the Majorana neutrino mass matrix can be expressed as,

$$M_\nu = V M^{\mathrm{diag}} V^T \tag{1}$$

where $M^{\mathrm{diag}} = \mathrm{diag}(m_1, m_2, m_3)$ is the diagonal matrix of neutrino masses and V is the flavor mixing matrix. The above equation can be re-written as

$$M_\nu = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T. \tag{2}$$

where $\lambda_1 = m_1 e^{2i\rho}, \lambda_2 = m_2 e^{2i\sigma}, \lambda_3 = m_3$. For our analysis, we consider the following parametrization of V [30]:

$$V \equiv UP = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} P, \tag{3}$$

where, $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$. Here, U is a 3×3 unitary matrix consisting of three flavor mixing angles (θ_{12}, θ_{23}, θ_{13}) and one Dirac CP-violating phase δ and $P = \mathrm{diag}(e^{2i\rho}, e^{2i\sigma}, 1)$, is a diagonal phase matrix consisting of two Majorana CP-violating phases ρ and σ.

If one of the elements of M_ν is considered zero, $i.e.$ $M_{lm} = 0$, we obtain the following constraint equation

$$\sum_{i=1,2,3} U_{li}U_{mi}\lambda_i = 0 \tag{4}$$

where l, m run over e, μ and τ. Two independent mass-squared differences δm^2 (solar) and Δm^2 (atmospheric) are defined as

$$\delta m^2 = (m_2^2 - m_1^2), \tag{5}$$

$$\Delta m^2 = |m_3^2 - m_2^2|. \tag{6}$$

The ratio of above mass-squared differences is given by

$$R_\nu = \frac{\delta m^2}{|\Delta m^2|} \, . \tag{7}$$

If we consider one of the neutrino masses to be zero then since, $m_2 > m_1$ has already been confirmed by solar neutrino oscillation data [31, 32], m_2 cannot be equal to zero. Thus, we are left with two possibilities where either m_1 or m_3 can vanish corresponding to normal ($m_1 = 0, m_2 < m_3$) or inverted ($m_1 < m_2, m_3 = 0$) mass ordering, respectively. The vanishing lowest neutrino mass along with one texture zero condition put constraints on the parameter space of neutrino masses, neutrino mixing angles and CP violating phases.

In case of one texture zero neutrino mass matrices, there exists a permutation symmetry between certain classes. This corresponds to permutation of the 2-3 rows and 2-3 columns of M_ν. The corresponding permutation matrix is

$$P_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \tag{8}$$

As a result of permutation symmetry between different classes, one obtains the following relations among the oscillation parameters

$$\theta_{12}^X = \theta_{12}^Y, \quad \theta_{23}^X = 90° - \theta_{23}^Y, \quad \theta_{13}^X = \theta_{13}^Y, \quad \delta^X = \delta^Y - 180°, \tag{9}$$

where X and Y denote the classes related by 2-3 permutation. The following one texture zero classes are related via permutation symmetry

$$P_2 \leftrightarrow P_3, \quad P_4 \leftrightarrow P_5 \, . \tag{10}$$

Classes P_1 and P_6 transform unto themselves under the action of P_{23}.

Case I: $m_1 = 0$ (normal mass ordering)

Using Eq. (4), we get the following expressions for the neutrino mass ratio $\left(\frac{m_2}{m_3}\right)$ and the Majorana phase σ

$$\frac{m_2}{m_3} = \frac{|U_{l3}U_{m3}|}{|U_{l2}U_{m2}|}, \tag{11}$$

$$\sigma = \frac{1}{2}\arg\left(-\frac{U_{l3}U_{m3}}{U_{l2}U_{m2}}\right). \tag{12}$$

Since m_1 is zero, therefore, Majorana phase ρ becomes unphysical in this case. Using Eqs. (5) and (6), neutrino masses (m_1, m_2, m_3) can be expressed in terms of experimentally known mass-squared differences ($\delta m^2, \Delta m^2$) as

$$m_1 = 0, \quad m_2 = \sqrt{\delta m^2}, \quad m_3 = \sqrt{\delta m^2 + \Delta m^2}. \tag{13}$$

Hence, we obtain

$$\frac{m_2}{m_3} = \sqrt{\frac{R_\nu}{1 + R_\nu}}. \tag{14}$$

Using Eqs. (11) and (14), we can express R_ν in terms of mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and Dirac CP-violating phase (δ) as

$$R_\nu = \left(\frac{|U_{l2}U_{m2}|^2}{|U_{l3}U_{m3}|^2} - 1\right)^{-1}. \tag{15}$$

Case II: $m_3 = 0$ (inverted mass ordering)

Parameter	Best Fit	1σ	2σ	3σ
$\delta m^2 \ [10^{-5}eV^2]$	7.60	7.42 - 7.79	7.26 - 7.99	7.11 - 8.18
$|\Delta m^2_{31}| \ [10^{-3}eV^2]$ (NO)	2.48	2.41 - 2.53	2.35 - 2.59	2.30 - 2.65
$|\Delta m^2_{31}| \ [10^{-3}eV^2]$ (IO)	2.38	2.32 - 2.43	2.26 - 2.48	2.20 - 2.54
θ_{12}	34.6°	33.6° - 35.6°	32.7° - 36.7°	31.8° - 37.8°
θ_{23} (NO)	48.9°	41.7° - 50.7°	40.0° - 52.1°	38.8° - 53.3°
θ_{23} (IO)	49.2°	46.9° - 50.7°	41.3° - 52.0°	39.4° - 53.1°
θ_{13} (NO)	8.6°	8.4° - 8.9°	8.2° - 9.1°	7.9° - 9.3°
θ_{13} (IO)	8.7°	8.5° - 8.9°	8.2° - 9.1°	8.0° - 9.4°
δ (NO)	254°	182° - 353°	0° - 360°	0° - 360°
δ (IO)	266°	210° - 322°	0° - 16° $\oplus$ 155° - 360°	0° - 360°

TABLE II. Current neutrino oscillation parameters from global fits at 1σ, 2σ and 3σ confidence level [6]. NO (IO) refers to normal (inverted) neutrino mass ordering.

The expressions for neutrino mass ratio $\left(\frac{m_2}{m_1}\right)$ and Majorana phase difference (ρ - σ) are given by

$$\frac{m_2}{m_1} = \frac{|U_{l1}U_{m1}|}{|U_{l2}U_{m2}|}, \tag{16}$$

$$\rho - \sigma = \frac{1}{2}arg\left(-\frac{U_{l2}U_{m2}}{U_{l1}U_{m1}}\right). \tag{17}$$

In the case of Inverted mass ordering, the phase difference ($\rho - \sigma$) is the relevant physical phase. From Eq. (17), it is clear that Majorana phases (ρ, σ) are linearly co-related. The neutrino mass spectrum for inverted mass ordering is given by

$$m_1 = \sqrt{\Delta m^2 - \delta m^2}, \quad m_2 = \sqrt{\Delta m^2}, \quad m_3 = 0. \tag{18}$$

The mass ratio $\left(\frac{m_2}{m_1}\right)$ is related to R_ν and is given by

$$\frac{m_2}{m_1} = \frac{1}{\sqrt{1 - R_\nu}}. \tag{19}$$

Using Eqs. (16) and (19), we can express R_ν in terms of mixing angles (θ_{12}, θ_{23}, θ_{13}) and Dirac CP-violating phase (δ) as

$$R_\nu = 1 - \left|\frac{U_{l2}U_{m2}}{U_{l1}U_{m1}}\right|^2. \tag{20}$$

The expression for Jarlskog rephasing parameter J_{CP}, which is a measure of CP violation, is given by

$$J_{CP} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta. \tag{21}$$

III.   NUMERICAL ANALYSIS

The experimental constraints on neutrino parameters at 1σ, 2σ and 3σ confidence level (CL) are given in Table II. The effective Majorana mass term relevant for neutrinoless double beta ($0\nu\beta\beta$) decay is given by

$$|M_{ee}| = |m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2|. \tag{22}$$

Observation of $0\nu\beta\beta$ decay will imply lepton number violation and Majorana nature of neutrinos. For reviews on $0\nu\beta\beta$ decay see Ref. [33, 34]. A large number of projects such as CUORICINO [35], CUORE [36], GERDA [37],

MAJORANA [38], SuperNEMO [39], EXO [40],GENIUS[41] aim to achieve a sensitivity upto 0.01eV for $|M_{ee}|$. We take the upper limit on $|M_{ee}|$ to be 0.5 eV [34]. Data from the Planck satellite [42] combined with other cosmological data put a limit on the sum of neutrino masses as

$$\Sigma = \sum_{i=1}^{3} m_i < 0.23\text{eV} \quad \text{at 95\% CL.} \tag{23}$$

In the present analysis, we assume a more conservative upper limit $\Sigma < 1$ eV, on the sum of neutrino masses.

Since we are considering one of the neutrino masses to be zero, we have two possibilities $i.e.$ either $m_1 = 0$ or $m_3 = 0$ corresponding to normal or inverted mass orderings, respectively. Eqs. (15) and (20) incorporate the constraints of a vanishing neutrino mass and one texture zero, for normal and inverted mass orderings, respectively. We span the parameter space of input neutrino oscillation parameters (θ_{12}, θ_{23}, θ_{13}, Δm^2, Δm^2) lying in their 3σ ranges by randomly generating points of the order of 10^7. Since the Dirac CP-violating phase δ is experimentally unconstrained at 3σ level, therefore, we vary δ within its full possible range $[0°, 360°]$. Using Eqs. (15) and (20) and the experimental inputs on neutrino mixing angles and mass-squared differences, the parameter space of δ, ρ, σ and $|M_{ee}|$ can be subsequently constrained.

It is found from our analysis that for one texture zero neutrino mass matrices with $m_1 = 0$, all the six patterns are now inconsistent with the latest global fits of neutrino oscillation data at 3σ level. This result is mainly due to the reduced 3σ errors on the experimentally measured values of θ_{13}. On the other hand, for singular one texture zero neutrino mass matrices with $m_3 = 0$, only Classes P_2, P_3, P_4, P_5 are found to be compatible with the latest experimental data at 3σ level. We have also performed the numerical analysis by considering 1σ and 2σ ranges of neutrino mixing angles and mass-squared differences.

<div align="center">Case I: $m_1 = 0$ (normal mass ordering)</div>

Classes P_2, P_3, P_6 with $m_1 = 0$ have already been ruled out in Ref. [17]. In the following we check the viability of the rest of the classes viz. P_1, P_4, P_5 with current experimental data.

Class P_1: Using Eq. (15), the exact analytical expression of R_ν in terms of θ_{13} is given by

$$R_\nu = \frac{t_{13}^4}{(s_{12}^4 - t_{13}^4)}, \tag{24}$$

where $t_{13} = \tan\theta_{13}$. With the experimentally allowed ranges of θ_{12} and θ_{13}, we find that R_ν turns out to be below its experimentally allowed 3σ range and hence is in conflict with the latest data.

Class P_4: With the help of Eq. (15), we obtain the following expression for R_ν in the leading order approximation of s_{13}

$$R_\nu \approx \frac{t_{23}^2 s_{13}^2}{s_{12}^2 c_{12}^2}. \tag{25}$$

The latest mixing data leads to rather higher values of R_ν, lying in the range (0.05 - 0.5) as compared to the allowed 3σ range (0.0279 - 0.0372) of R_ν and hence class P_4 is excluded by current experimental data at 3σ CL.

Class P_5 : We obtain the following expression for R_ν in the leading order approximation of s_{13}

$$R_\nu \approx \frac{s_{13}^2}{t_{23}^2 s_{12}^2 c_{12}^2}. \tag{26}$$

Since classes P_5 and P_4 are related via permutation symmetry given in Eq. (8), their phenomenological implications are similar. As in the case of class P_4 the latest mixing data leads to values of R_ν above its experimentally allowed 3σ range hence, class P_5 is also ruled out by the latest experimental data. Ref. [28] has also found classes P_4 and P_5 to be incompatible with the recent data for normal mass ordering. Further, in Ref. [29] it has been shown that these classes remain disfavoured even after taking into account the renormalization group effects.

Classes P_4 and P_5 with normal mass ordering have been studied in Ref. [20] with the additional constraint of CSD2 [20, 21]. In Ref. [22], classes P_1, P_4 and P_5 with normal mass ordering have been examined with the outcome that the (1,3) element of the neutrino mixing matrix and the parameter R_ν are found to be linked for these classes.

All these classes are now incompatible with the recent data [6] for the normal mass ordering and one has to consider modifications to these classes for them to be compatible with the latest data. In this direction, small charged lepton contributions to classes P_4 and P_5 along with the CSD2 constraint have been considered in Ref. [21] where, a unified indirect family symmetry model has been constructed, which leads to $\theta_{13} \sim 8°$ - $9°$.

Case II: $m_3 = 0$ (inverted mass ordering)

For the inverted neutrino mass ordering case, classes P_2, P_3, P_4, P_5 are found to be compatible with the latest neutrino oscillation data at 3σ CL. Interestingly, classes P_2 and P_3 cannot satisfy the experimental data at 1σ CL while remaining classes P_4 and P_5 predict the Dirac phase δ to be near $270°$ at the same confidence level. The correlation plots for these classes are given in figures 1 - 3 using 3σ ranges of the known neutrino oscillation parameters while numerical results at 1σ, 2σ and 3σ CL are given in Table II. Discussed below are the phenomenological implications of the experimentally allowed classes.

Class P_2: To the leading order in s_{13} we get the following expression for the mass ratio $\left(\frac{m_2}{m_1}\right)$

$$\frac{m_2}{m_1} \approx t_{12}^2 \left(1 + \frac{2c_\delta s_{13} t_{23}}{s_{12} c_{12}}\right), \tag{27}$$

Some of the interesting plots for class P_2 are shown in Figs. 1 and 2. In Fig. 1(a) we have shown the correlation plot between the two Majorana phases ρ and σ. In the case of $m_3 = 0$, one has the freedom to make an overall phase rotation of neutrino masses so that the Majorana phase associated with one of the non-zero neutrino masses *i.e.* m_1 or m_2 may be rotated away and we are left with only one phase difference which is physical. In Fig. 1(b) we have shown the correlation plot between the physical Majorana phase difference $(\rho - \sigma)$ and Dirac phase δ. One can see that $(\rho - \sigma)$ and δ are constrained to small ranges for class P_2. A vanishing δ is still possible for class P_2 which allows for vanishing J_{CP} along with non-zero values [Fig. 1(c)]. The effective Majorana mass $|M_{ee}|$ has been plotted against δ in Fig. 1(d). It is clear that only a narrow range for $|M_{ee}| \sim (0.0102$ - $0.0205)$ eV is allowed for class P_2.
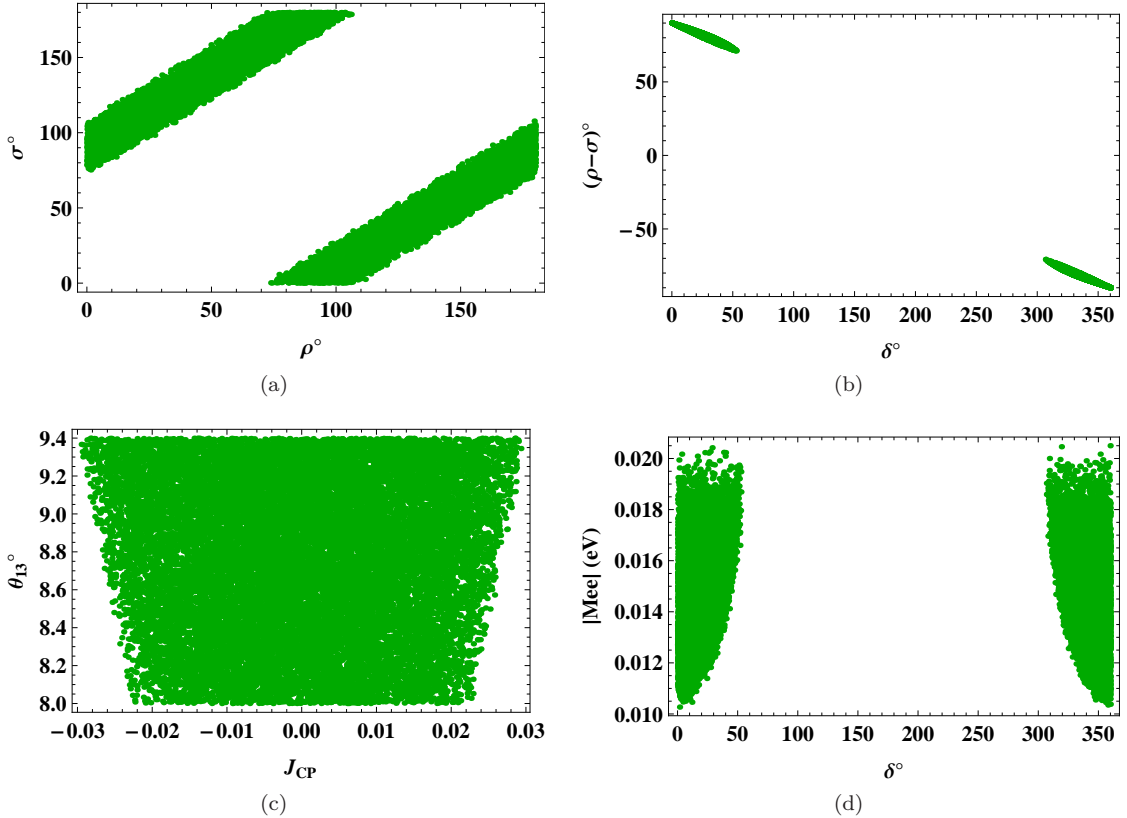

FIG. 1. Correlation plots for class P_2

Class P_3: Since class P_3 is related to class P_2 via permutation symmetry [Eq. (8)], the phenomenological implications for class P_3 can be obtained from class P_2 using Eq. (9). The 2-3 interchange symmetry between classes P_2
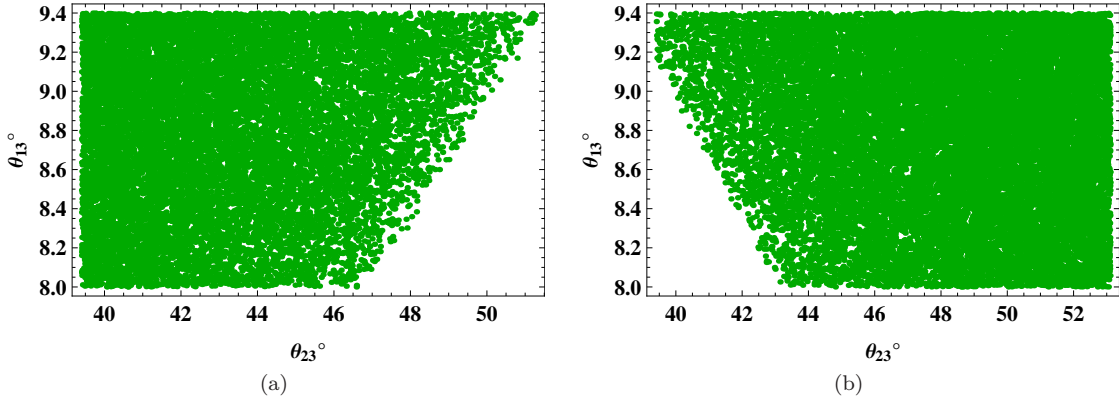
FIG. 2. Correlation plots for classes P_3 (a) and P_2 (b) depicting the 2-3 interchange symmetry.

and P_3 is shown in figure 2. From Fig. 2(a) (2(b)) one can see that for higher values of θ_{13}, lower (upper) quadrant of θ_{23} is preferred for class P_3 (P_2). Figs. 2(a) and (b) may appear to show slight deviation from the 2-3 interchange symmetry relation:

$$\theta_{23}^{P_3} = 90° - \theta_{23}^{P_2}. \tag{28}$$

However, this apparent deviation is just because the experimentally allowed 3σ range for θ_{23} is not symmetric around $\theta_{23} = 45°$.

Class P_4: We have the following expression for the mass ratio $\left(\frac{m_2}{m_1}\right)$ in leading order terms of s_{13}

$$\frac{m_2}{m_1} \approx \left(1 + \frac{c_\delta s_{13} t_{23}}{s_{12} c_{12}}\right). \tag{29}$$

The correlation plots for class P_4 have been compiled in figure 3. Fig. 3(a) shows the correlation plot between the Majorana phases ρ and σ. The physically relevant phase difference $(\rho - \sigma)$ has been plotted against Dirac phase δ in Fig. 3(b). Both $(\rho - \sigma)$ and δ have very small allowed ranges for class P_4. The allowed parameter space for δ is constrained near $90°$ and $270°$, which leads to the result that J_{CP} cannot vanish for this class [Fig. 3(c)]. A very narrow range for $|M_{ee}| \sim (0.0412 - 0.0520)$ eV is allowed for class P_4 [Fig. 3(d)].

Class P_5: As class P_5 is related to class P_4 via permutation symmetry [Eq. (8)], the phenomenological implications for class P_5 can be obtained from class P_4 using Eq. (9). The allowed values of $|M_{ee}|$ for classes P_4 and P_5 are on the larger side having no overlap with classes P_2 and P_3 [Table III]. Thus, $|M_{ee}|$ can be used to distinguish between diagonal and off-diagonal one texture zero classes with a vanishing neutrino mass.
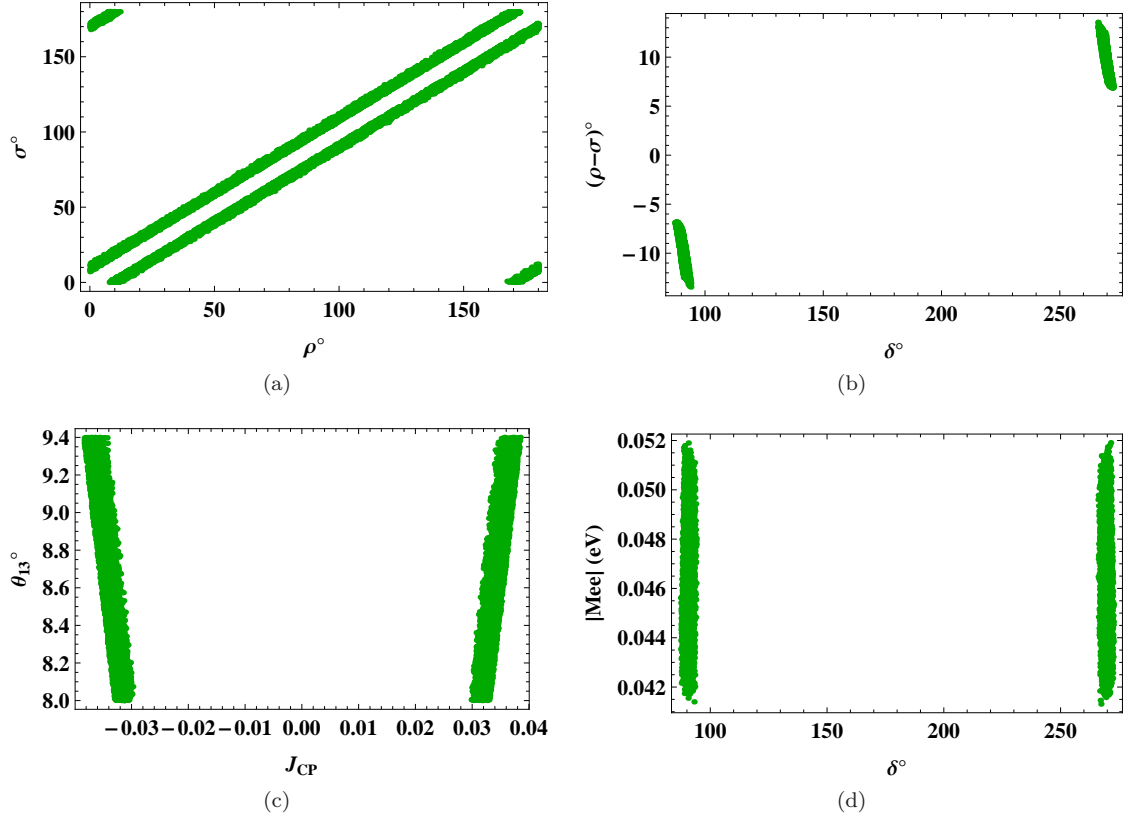
IV. SYMMETRY REALIZATION

Singular one texture zero neutrino mass matrices can be realized using a discrete Abelian flavor symmetry within the context of type-I seesaw mechanism [26]. Such texture structures have been realized earlier in Ref.[17] using $Z_{12} \times Z_2$ symmetry, requiring six (seven) $SU(2)_L$ doublet Higgses for the classes where the texture zero is present on the diagonal (off-diagonal) elements. Here, we show how the phenomenologically allowed singular one texture zero classes can be realized with a much smaller number of Higgs doublets and a smaller symmetry group. For the classes where the texture zero corresponds to diagonal elements we need only two Higgs doublets and for the off-diagonal texture zero classes only three Higgs doublets are required. The symmetry group used to realize these texture structures is Z_8.

Within the framework of type-I seesaw mechanism the effective neutrino mass matrix is given by

$$M_\nu \approx M_D M_R^{-1} M_D^T \tag{30}$$

where M_D and M_R are the Dirac and the right-handed neutrino mass matrices, respectively. To realize the texture structures within the framework of type-I seesaw mechanism we extend the standard model by adding three right-handed neutrinos ($\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$) and a $SU(2)_L$ singlet scalar (χ).

FIG. 3. Correlation plots for class P_4

Class	CL	δ	$\rho - \sigma$	$|M_{ee}|$(eV)	J_{CP}
P_1	1σ	×	×	×	×
	2σ	×	×	×	×
	3σ	×	×	×	×
P_2	1σ	×	×	×	×
	2σ	$0° - 16° \oplus 317° - 360°$	$(-90.00°) - (-74.80°) \oplus 83.82° - 90.00°$	0.0122 - 0.0187	$(-0.0240) - 0.0102$
	3σ	$0° - 53° \oplus 306° - 360°$	$(-90.0°) - (-70.5°) \oplus 71.0° - 90.0°$	0.0102 - 0.0205	$(-0.03) - 0.03$
P_3	1σ	×	×	×	×
	2σ	$154.7° - 214.0°$	$(-90.0°) - (-81.5°) \oplus 78.5° - 90.0°$	0.0123 - 0.0167	$(-0.0206) - 0.0156$
	3σ	$130° - 230°$	$(-90°) - (-73°) \oplus 73° - 90°$	0.0102 - 0.0187	$(-0.029) - 0.029$
P_4	1σ	$267.8° - 269.9°$	$9.45° - 11.55°$	0.0446 - 0.0482	$(-0.0356) - (-0.0327)$
	2σ	$266.8° - 271.7°$	$7.4° - 12.5°$	0.0428 - 0.0501	$(-0.0369) - (-0.0308)$
	3σ	$87.2° - 94.2° \oplus 265.9° - 272.9°$	$(-13.5°) - (-6.7°) \oplus 6.7° - 13.5°$	0.0412 - 0.0520	$(-0.0385) - (-0.0295) \oplus 0.0295 - 0.0385$
P_5	1σ	$268.5° - 270.6°$	$(-8.90°) - (-7.26°)$	0.0449 - 0.0485	$(-0.0560) - (-0.0327)$
	2σ	$267.6° - 272.3°$	$(-11.2°) - (-6.6°)$	0.0431 - 0.0502	$(-0.0369) - (-0.0309)$
	3σ	$86.5° - 93.5° \oplus 266.5° - 273.5°$	$(-12.5°) - (-6°) \oplus 6° - 12.5°$	0.0413 - 0.0521	$(-0.0385) - (-0.0295) \oplus 0.0295 - 0.0385$
P_6	1σ	×	×	×	×
	2σ	×	×	×	×
	3σ	×	×	×	×

TABLE III. The allowed ranges of Dirac CP-violating phase δ, the physical Majorana phase difference $(\rho - \sigma)$, effective Majorana mass $|M_{ee}|$ and Jarlskog rephrasing invariant J_{CP} for the experimentally allowed classes with $m_3 = 0$.

For Illustration, we show in detail the symmetry realization of class P_2 where the (2,2) element of M_ν is zero. For

class P_2 we assume the following transformation properties of the leptonic fields under the cyclic group Z_8

$$\overline{D}_{eL} \to \omega^7 \overline{D}_{eL}, \qquad e_R \to \omega e_R, \qquad \nu_{eR} \to \nu_{eR},$$
$$\overline{D}_{\mu L} \to \omega^4 \overline{D}_{\mu L}, \qquad \mu_R \to \omega^4 \mu_R, \qquad \nu_{\mu R} \to \omega^4 \nu_{\mu R}, \qquad (31)$$
$$\overline{D}_{\tau L} \to \omega^5 \overline{D}_{\tau L}, \qquad \tau_R \to \omega^5 \tau_R, \qquad \nu_{\tau R} \to \omega \nu_{\tau R},$$

where $\omega = e^{i2\pi/8}$, D_{fL} $(f = e, \mu, \tau)$ denote $SU(2)_L$ doublets and l_{fR}, ν_{fR} denote the right-handed $SU(2)_L$ singlet charged lepton and neutrino fields, respectively. According to the above transformations of the leptonic fields, the bilinears $\overline{D}_{fL} l_{gR}$, $\overline{D}_{fL} \nu_{gR}$ and $\nu_{fR}^T C^{-1} \nu_{gR}$ relevant for M_l, M_D and M_R, respectively, transform as

$$\overline{D}_{fL} l_{gR} \sim \begin{pmatrix} 1 & \omega^3 & \omega^4 \\ \omega^5 & 1 & \omega \\ \omega^6 & \omega & \omega^2 \end{pmatrix}, \quad \overline{D}_{fL} \nu_{gR} \sim \begin{pmatrix} \omega^7 & \omega^3 & 1 \\ \omega^4 & 1 & \omega^5 \\ \omega^5 & \omega & \omega^6 \end{pmatrix}, \quad \nu_{fR}^T C^{-1} \nu_{gR} \sim \begin{pmatrix} 1 & \omega^4 & \omega \\ \omega^4 & 1 & \omega^5 \\ \omega & \omega^5 & \omega^2 \end{pmatrix}. \quad (32)$$

We introduce two $SU(2)_L$ doublet Higgs (ϕ_1, ϕ_2) transforming as: $\phi_1 \to \phi_1$ and $\phi_2 \to \omega^6 \phi_2$ under the action of Z_8. These transformation properties of ϕ_1 and ϕ_2 will lead to the following Z_8 invariant Yukawa Lagrangian for class P_2

$$-\mathcal{L}_Y = Y_1(\overline{D}_{eL} e_R)\phi_1 + Y_2(\overline{D}_{\mu L} \mu_R)\phi_1 + Y_3(\overline{D}_{\tau L} \tau_R)\phi_2 + Y_4(\overline{D}_{eL} \nu_{\tau R})\tilde{\phi}_1$$
$$+ Y_5(\overline{D}_{\mu L} \nu_{\mu R})\tilde{\phi}_1 + Y_6(\overline{D}_{\tau L} \nu_{\tau R})\tilde{\phi}_2 + \text{H. c.} \quad (33)$$

where $\tilde{\phi}_j = i\tau_2 \phi_j^*$ $(j = 1, 2)$. When the Higgs fields (ϕ_j) acquire non-zero vacuum expectation values $\langle\phi_j\rangle_o \neq 0$, we get the charged lepton mass matrix M_l and the Dirac neutrino mass matrix M_D of the following form

$$M_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad (34)$$

$$M_D = \begin{pmatrix} 0 & 0 & a \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}, \quad (35)$$

where $m_e = Y_1\langle\phi_1\rangle_o$, $m_\mu = Y_2\langle\phi_1\rangle_o$, $m_\tau = Y_3\langle\phi_2\rangle_o$, $a = Y_4\langle\phi_1^*\rangle_o$, $b = Y_5\langle\phi_1^*\rangle_o$ and $c = Y_6\langle\phi_2^*\rangle_o$. For the right-handed Majorana neutrino mass matrix, we assume a $SU(2)_L$ singlet scalar (χ) transforming as $\chi \to \omega^3 \chi$ under Z_8, thus χ will lead to non-zero (2,3) element of M_R. Also, non-zero (1,1) and (2,2) elements of M_R arise from bare Majorana mass terms which are already invariant under Z_8. This leads to the following form of M_R

$$M_R = \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & 0 \end{pmatrix}. \quad (36)$$

Using the type-I seesaw mechanism, these M_D and M_R lead to an effective neutrino mass matrix having a zero (2,2) element and a vanishing neutrino mass (class P_2).

By assigning suitable transformation properties to the leptonic and Higgs fields under the action of Z_8, one can realize the remaining phenomenologically allowed classes of singular neutrino mass matrices with one texture zero. The structures of M_D and M_R for all the phenomenologically allowed classes are summarized in Table IV. The leptonic and Higgs field transformation properties under Z_8, leading to all the viable one texture zero classes with a vanishing neutrino mass are given in Table V.

V. SUMMARY

We have done a systematic analysis of all the one texture zero neutrino mass matrices with a vanishing neutrino mass using the latest global fits of neutrino oscillation parameters. We find that all the six classes with normal mass ordering are now ruled out at 3σ confidence level whereas in case of inverted mass ordering only four classes (P_2, P_3, P_4, P_5) out of total six are consistent with the latest experimental data. Furthermore, only classes P_4 and P_5 are found to be compatible with the latest data at 1σ confidence level. For classes P_4 and P_5 with inverted mass ordering, the parameter space of Dirac CP-violating phase δ is restricted to values near $\delta \approx 90°$, $270°$ $(\delta \approx 270^0)$ at

Class	M_D	M_R
P_2	$\begin{pmatrix} 0 & 0 & a \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$	$\begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & 0 \end{pmatrix}$
P_3	$\begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ 0 & 0 & c \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & A \\ 0 & B & 0 \\ A & 0 & C \end{pmatrix}$
P_4	$\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ c & 0 & d \end{pmatrix}$	$\begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}$
P_5	$\begin{pmatrix} a & 0 & 0 \\ b & 0 & c \\ 0 & 0 & d \end{pmatrix}$	$\begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix}$

TABLE IV. Structures of M_D and M_R leading to experimentally allowed singular one texture zero effective neutrino mass matrices.

Class	$\overline{D}_{eL},\ \overline{D}_{\mu L},\ \overline{D}_{\tau L}$	$e_R,\ \mu_R,\ \tau_R$	$\nu_{eR},\ \nu_{\mu R},\ \nu_{\tau R}$	ϕ_1	ϕ_2	ϕ_3	χ
P_2	$\omega^7,\ \omega^4,\ \omega^5$	$\omega,\ \omega^4,\ \omega^5$	$1,\ \omega^4,\ \omega$	1	ω^6	$-$	ω^3
P_3	$\omega^7,\ \omega^5,\ \omega^4$	$\omega,\ \omega^5,\ \omega^4$	$\omega,\ 1,\ \omega^4$	1	ω^6	$-$	ω^3
P_4	$1,\ \omega^6,\ \omega^3$	$1,\ \omega,\ \omega^4$	$1,\ \omega^4,\ \omega^5$	1	ω	ω^3	ω^6
P_5	$1,\ \omega^3,\ \omega^6$	$1,\ \omega^4,\ \omega$	$1,\ \omega^4,\ \omega^5$	1	ω	ω^3	ω^6

TABLE V. Transformation properties of lepton and scalar fields under Z_8 for classes P_2, P_3, P_4 and P_5.

3σ (1σ) confidence level. We have shown how the experimentally allowed classes can be realized within the context of type-I seesaw mechanism using Z_8 discrete symmetry and a small number of Higgs doublets. Classes where the texture zero corresponds to diagonal elements can be distinguished from the off-diagonal one texture zero classes on the basis of allowed 3σ ranges of effective Majorana mass for these classes. For all the experimentally allowed classes we get ranges for the effective Majorana mass, which lie within the sensitivity limits of future neutrinoless double beta decay experiments.

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